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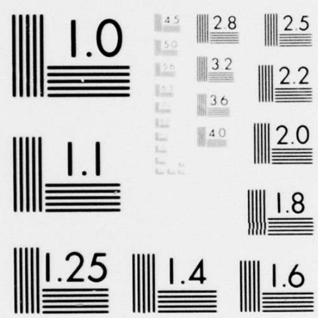
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A Note on Covariance-Invariant Digital Filter Design

and

Autoregressive Moving Average Spectrum Analysis

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ONR Technical Report #22

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A Note on Covariance-Invariant Digital Filter Design

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Autoregressive Moving Average Spectrum Analysis

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Manuscript submitted as a Correspondence Item

to the

IEEE Trans. on Acoustics, Speech, and Signal Processing

November 1978

This work partially supported by the Office of Naval Research, Statistics and Probability Branch, Arlington, VA, under Contract N00014-75-C-0518.

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Abstract

Consider an autoregressive-moving average (ARMA) discrete-time sequence $\{x_k\}$ with covariance sequence $\{R_k\}$. Equations are given for the solution of the AR coefficients $\{a_k\}_1^n$ in terms of the covariance $\{R_k\}_0^{2n-1}$, and subsequent solution for the MA coefficients $\{b_k\}_1^n$ in terms of the AR coefficients and the covariances $\{R_k\}_0^{n-1}$. The results are derived and presented somewhat differently than usual to complement the results of [1] for the synthesis of covariance-invariant digital filters. In the context of spectrum analysis, the results provide a means of performing ARMA spectrum analysis on data that arise as sampled data from a rational continuous-time process [2]. An important result, originally derived in [2], shows that the ARMA spectrum can be obtained without actually solving the nonlinear factorization problem for the MA coefficients.

I. Introduction

Consider a zero-mean wide-sense stationary process $x(t)$ generated as the output of an nth-order continuous-time filter $H_c(s)$ excited by white noise and with covariance function $R_c(\tau)$. A discrete-time sequence $\{x_k\}_0^\infty$ with covariance sequence $\{R_k\}$ is said to be covariance-invariant with $x(t)$ if $R_k = R_c(\tau=kT)$ for $k = 0, \pm 1, \pm 2, \ldots$. The sequence $\{x_k\}$ may be obtained as the output of an ARMA (n,n-1) discrete-time filter $H(z)$. $H(z)$ is thus said to be covariance-invariant with $H_c(s)$ [1], [2].

Covariance-invariant filters arise in a number of contexts, including signal processing applications [1], ARMA spectrum estimation [2], and speech analysis. The problem is to synthesize the discrete-time filter $H(z)$ from knowledge only of the $\{R_k\}_0^{2n-1}$. When synthesizing covariance-invariant digital filters, the $\{R_k\}_0^{2n-1}$ are characterized by $R_k = R_c(\tau=kT)$ with $R_c(\tau)$ obtained from a given transfer function $H_c(s)$ as outlined in [1], [2], or [10]. In spectrum and speech analysis applications, the $\{R_k\}_0^{2n-1}$ are replaced by estimated covariances as discussed in [2].

II. Some Basic Results for Discrete-Time Systems

Every nth-order continuous-time filter $H_c(s)$ has a covariance-invariant $H(z)$ that is ARMA (n,n-1) [1], [2]. Therefore, it suffices to consider transfer functions of the form

$$H(z) = \frac{b_1 z^{-1} + b_2 z^{-2} + \ldots + b_n z^{-n}}{a_0 + a_1 z^{-1} + \ldots + a_n z^{-n}} \; ; \quad a_0 = 1 \tag{1}$$

This is the transfer function of an ARMA (n,n-1) discrete-time filter. It is termed strictly proper because $\lim_{z\to\infty} H(z) = 0$. The filter $zH(z)$ has the same spectrum as $H(z)$, but is only proper; i.e., $\lim_{z\to\infty} zH(z) < \infty$. Consideration of strictly proper transfer functions simplifies the state-space model of interest to us, so that we assume a transfer function of the form (1) throughout.

The difference equation corresponding to (1) is

$$x_k + \sum_{\ell=1}^{n} a_\ell x_{k-\ell} = \sum_{\ell=1}^{n} b_\ell \, u_{k-\ell} \tag{2}$$

where $\{u_k\}$ and $\{x_k\}$ are, respectively, input and output sequences. The unit pulse response of $H(z)$ is $\{h_k\}$ where:

$$h_k = \begin{cases} 0, & k \leq 0 \\ b_k - \sum_{\ell=1}^{n} a_\ell h_{k-\ell} \, , & k > 0 \end{cases} \tag{3}$$

The companion form state model for (2) is [4]

$$X_{k+1} = \phi \, X_k + \bar{h} u_k$$

$$x_k = \Psi \, X_k \; ; \quad \Psi = (1 \; 0 \ldots \; 0)$$

$$\phi = \begin{bmatrix} \bar{0} & I \\ -a_1 -a_2 \ldots -a_n \end{bmatrix} \qquad \bar{h}' = (h_1 \ldots h_n) \tag{4}$$

$\bar{0}$ an (n-1)x1 vector of zeros and I an (n-1)x(n-1) identify matrix. Initial conditions follow from (3) and prime denotes transpose.

The characteristic equation for ϕ is

$$\lambda^n + a_1\lambda^{n-1} + \ldots + a_{n-1}\lambda + a_n = 0 \qquad (5)$$

The Cayley-Hamilton theorem says ϕ satisfies its own characteristic equation:

$$\sum_{\ell=0}^{n} a_\ell \, \phi^{n-\ell} = 0 \quad ; \quad a_0 = 1 \qquad (6)$$

III. Covariance Results

Now assume $\{u_k\}$ is a unit-variance, zero-mean white sequence. For ϕ a stability matrix, the steady-state covariance of $\{x_k\}_0^\infty$ is simply

$$R_k = (\phi^k V_d)_{11} \; , \; k = 0, 1, 2, \ldots \qquad (7)$$

where $(M)_{11}$ denotes the (1,1) element of M. The matrix V_d is the steady-state covariance of the state X_k 4 : $V_d = \phi V_d \phi' \bar{h}\bar{h}'$.

For $k \geq n$, we may write

$$\sum_{m=0}^{n} a_m R_{k-m} = (\phi^{k-n} \sum_{m=0}^{n} a_m \phi^{n-m} V_d)_{11} \quad , \quad k \geq n \qquad (8)$$

But from the Cayley-Hamilton theorem, the RHS of (8) is zero. Therefore,

$$\sum_{m=0}^{n} a_m R_{k-m} = 0 \quad , \quad k \geq n \qquad (9)$$

That is, for covariance lags greater than or equal to n, an ARMA (n,n-1) system "looks purely autoregressive." [2], [5], [6], [7].

An alternative expression for the covariance sequence $\{R_k\}_0^\infty$ is

$$R_k = R_{-k} = \sum_{m=1}^{\infty} h_m h_{m+k} \quad k = 0, 1, 2, \ldots \tag{10}$$

Substitution of (3) for h_{m+k} and a modest amount of algebra leads to the following result:

$$\sum_{\ell=0}^{n} a_\ell R_{k-\ell} = \sum_{m=1}^{n-k} h_m b_{m+k} \quad , \quad k = 0, 1, 2, \ldots \tag{11}$$

For $k \geq n$ this gives the same result as (9). Note also that when $b_m = 0$ for $m > 1$ (an AR system), the RHS of (11) is b_1^2. Thus (11) is a general set of normal equations that includes the AR normal equations as a special case.

IV. The Moving Average Coefficients

To obtain an expression for the MA coefficients $\{b_k\}_1^n$ in terms of the AR coefficients $\{a_k\}_0^n$ and the covariances $\{R_k\}_0^{n-1}$, we write out (11) for $k = n-1, n-2, \ldots, 1, 0$, use the result of (3) for h_k, and substitute from top to bottom. We thus obtain the following system of nonlinear equations in the MA coefficients $\{b_k\}_1^n$:

$$\sum_{\ell=1}^{n-k} b_\ell b_{\ell+k} = r_k \quad , \quad k = 0, 1, \ldots, n-1$$

$$r_k = \sum_{m=0}^{n-k-1} a_m \sum_{\ell=0}^{n} a_\ell R_{k+m-\ell} \tag{12}$$

The expression for the $\{r_k\}$ may alternatively be written

$$r_k = \sum_{m=0}^{n} a_m \sum_{\ell=0}^{n} a_\ell R_{k+m-\ell} \tag{13}$$

This is the expression one obtains for the RHS of (12) by proceeding directly from (2) to derive an expression for $\sum_{\ell=1}^{n-k} b_\ell b_{\ell+k}$. Equations (12) and (13) are entirely equivalent. The extra terms in (13) vanish by virtue of the linear dependence established in (9).

There is a spectral factorization problem imbedded in the equations of (12). Write

$$\sum_{\ell=-\infty}^{\infty} b_\ell b_{\ell+k} = r_k \quad , \quad k = 0, \pm 1, \pm 2, \ldots \tag{14}$$

with $b_\ell = 0$ for $\ell \leq 0$ and $\ell \geq n+1$, $r_k = 0$ for $|k| \geq n$, and $r_k = r_{-k}$.

Note $\{r_k\}$ is simply the covariance of the residual sequence $\{\sum_{\ell=0}^{n} a_\ell x_{k-\ell}\}$. Z-transform (14) to obtain

$$B(z)B(z^{-1}) = S(z) \tag{15}$$

with

$$B(z) = \sum_{k=1}^{n} b_k z^{-k} = z^{-n} \sum_{k=1}^{n} b_k z^{n-k}$$

$$S(z) = \sum_{k=-(n-1)}^{n-1} r_k z^{-k} = r_0 + \sum_{k=1}^{n-1} r_k (z_k^{-1} + z_k) \tag{16}$$

When the spectrum $S(z)$ is factored for $B(z)$, a polynomial of the following form is obtained

$$B(z) = Kz^{-n} \prod_{k=1}^{n-1} (z-z_k) \tag{17}$$

But $(B(z)B(z^{-1})$ is a mirror image polynomial, as the result $S(z) = S(z^{-1})$ shows. Therefore, one may as well use for $B(z)$ the expression of (17) with an arbitrary number of the z_k replaced by z_k^{-1}. Thus there are $M = 2^{n-1}$ different solutions for $B(z)$ and, correspondingly, 2^{n-1} different sets of MA coefficients one may choose. The unique set for which $B(z)$ has all its zeros inside the unit circle is the minimum phase set. There may be other sets, such as the set with all zeros outside the unit circle, that have more desirable "phase properties."

V. Application to Covariance-Invariant Digital Filter Design

The preceding results are applied to the synthesis of covariance-invariant digital filters as follows:

(1) Begin with a stable continuous-time transfer function $H_c(s)$ and solve for the corresponding covariance function $R_c(\tau)$. See [1], [2], and [10] for useful expressions. Set $R_k = R_c(\tau=kT)$.

(2) Write out (9) for k=n, n+1, ..., 2n-1 and solve the resulting Toeplitz set of linear equations for the AR parameters $\{a_k\}_1^n$. See [7] and [8] for references to related equations and software for efficiently solving such equations. See [7] for a more general discussion of the related "normal equations" that arise in autoregressive or linear prediction of speech.

(3) Solve either (12) or (15) for the MA coefficients $\{b_k\}_1^n$. If (12) is solved, then a polynomial $B(z)$ may be formed from the $\{b_k\}_1^n$ and factored for its roots. Roots lying outside the unit circle may be reflected inside to obtain a variety of phase functions, ranging from minimum to maximum phase.

VI. Spectrum Analysis

Consider the power spectrum $S_d(f)$ corresponding to the filter $H(z)$:

$$S_d(f) = H(z=e^{j2\pi fT})H(z^{-1}=e^{-j2\pi fT})$$

$$= \frac{\sum_{k=1}^{n}\sum_{\ell=1}^{n} b_k b_\ell e^{j2\pi(\ell-k)fT}}{\sum_{k=0}^{n}\sum_{\ell=0}^{n} a_k a_\ell e^{j2\pi(\ell-k)fT}} \tag{18}$$

The double sums in the numerator and denominator may be evaluated on upper and lower triangular grids to write

$$S_d(f) = \frac{\sum_{k=1}^{n} b_k^2 + 2\sum_{\ell=1}^{n-1}\sum_{k=1}^{n-\ell} b_k b_{k+\ell}\cos 2\pi\ell fT}{\sum_{k=0}^{n} a_k^2 + 2\sum_{k=1}^{n} a_k \cos 2\pi kfT + 2\sum_{\ell=1}^{n}\sum_{k=1}^{n-\ell} a_k a_{k+\ell}\cos 2\pi\ell fT} \tag{19}$$

From (12) we may write

$$S_d(f) = \frac{r_0 + 2\sum_{\ell=1}^{n-1} r_\ell \cos 2\pi\ell fT}{\sum_{k=0}^{n} a_k^2 + 2\sum_{k=1}^{n} a_k \cos 2\pi kfT + 2\sum_{\ell=1}^{n}\sum_{k=1}^{n-\ell} a_k a_{k+\ell}\cos 2\pi\ell fT} \tag{20}$$

This result shows that the ARMA spectrum may be found in terms of the AR parameters $\{a_\ell\}_0^n$, and the residual covariance $\{r_\ell\}_0^{n-1}$, without ever solving for the MA coefficients. Thus, a nonlinear spectral factorization problem is solved.

VII. Concluding Remarks

When the results discussed here are used for spectrum analysis, then the covariance $\{R_k\}_0^{2n-1}$ are replaced everywhere by estimated variables $\{\hat{R}_k\}_0^{2n-1}$. Gersch [6] has shown that when the order n is known, and covariances are estimated as $\hat{R}_k = N^{-1} \sum_{\ell=1}^{N} x_\ell x_{\ell+k}$, then the AR coefficients found from the normal equations are unbiased and consistent. It does not follow, however, that for finite N the AR coefficients will correspond to a stable filter H(z). That is, the equations of (9) do not enjoy the nice stability property associated with the normal equations

$$\sum_{m=0}^{n} a_m R_{k-m} = 0 \text{ , } k = 1, 2, \ldots, n$$

This defect may not be critical from the point of view of spectrum analysis. If a stable filter is desired, unstable poles of H(z) may be reflected inside the unit circle. A more substantial defect is that the estimated residual sequence $\{r_k\}$ may not be a true covariance sequence. This means the numerator of (20) may be negative for some f. Thus, care must be taken when using (20) to ensure that a negative spectrum is not estimated. This consideration is but a subset of the more general problem of order determination in AR and ARMA spectrum estimation. A thorough study of order determination in ARMA (n,n-1) spectrum analysis is required before such analysis becomes a well developed tool. Nonetheless, we feel ARMA (n,n-1) spectrum analysis is the natural approach when dealing with data that arise as sampled data from a rational continuous-time process. The calculations are not markedly more complicated than for AR (n) spectrum analysis.

See [6] for a more complete list of references to the statistics literature dealing with ARMA time series.

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ONR Technical Report #22	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A Note on Covariance-Invariant Digital Filter Design and Autoregressive Moving Average Spectrum Analysis		5. TYPE OF REPORT & PERIOD COVERED Technical Report 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John F. Kinkel, Joseph Perl, Louis L. Scharf, and Allen R. Stubberud		8. CONTRACT OR GRANT NUMBER(s) N00014-75-C-0518
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Electrical Engineering Colorado State University Fort Collins, CO 80523		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Statistics and Probability Branch Arlington, VA 22217		12. REPORT DATE November 1978 13. NUMBER OF PAGES 14
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Office of Naval Research Statistics and Probability Branch Arlington, VA 22217		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Autoregressive moving average time series, spectral analysis digital filtering, covariance-invariant spectral factorization

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Consider an autoregressive-moving average (ARMA) discrete-time sequence $\{x_k\}$ with covariance sequence $\{R_k\}$. Equations are given for the solution of the AR coefficients $\{a_k\}_1^n$ in terms of the covariance $\{R_k\}_0^{2n-1}$, and subsequent solution for the MA coefficients $\{b_k\}_1^n$ in terms of the AR coefficients and the covariances $\{R_k\}_0^{n-1}$. The results are derived and presented somewhat differently than usual to complement the results of ~~[1]~~ for the synthesis of covariance-invariant digital filters. In the context of spectrum analysis, the results provide a means of performing ARMA spectrum analysis on data that arise as

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S/N 0102-014-6601

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sampled data from a rational continuous-time process ~~[2]~~. An important result, originally derived in [2], shows that the ARMA spectrum can be obtained without actually solving the nonlinear factorization problem for the MA coefficients.